FINDING THE CONVEX HULL

- Introduction
- Methods for finding convex hull
- Package wrapping
- Graham’s scan method
- Interior elimination
- Conclusion
INTRODUCTION

- Convex polygon
  A polygon in which any line connecting 2 points inside the polygon must lie inside the polygon
- Convex hull of a given set of points
  The smallest convex polygon encompassing all the points
- A convex hull can contain as small as 3 points or as many as all the points
METHODS OF FINDING CONVEX HULL

- Package wrapping
- Graham’s scan
- Interior elimination
PACKAGE WRAPPING

- Called package wrapping because conceptually it is similar to wrapping a package
- Starts from the point having the smallest y coordinate
- Closely mirrors how a human would draw a convex hull
ALGORITHM

- Start from the point having smallest y (guaranteed to be on the convex hull)
- Take a horizontal ray starting from min y and sweep it upward (i.e., in counterclockwise direction).
- On encountering a point check to see if the point found is the first anchor (the smallest y point)
  - if yes then quit
  - else
    - Put the point on the hull, set this point as the current anchor. Continue the sweeping process from the horizontal ray with the current anchor.
Implementation

- ABCDEFGHIJKLMNOP
- BACDEFGHIJKLMNOP
- BMCDDEFGHIJKLMNOP
- BMLEDDEFGHIJKCANOPL
- BMLNEDDEFGHIJKCADOP

Angle:
Take the current anchor as the starting point and the remaining points as the ending points to form rays.
“Angle” is the degree swept through Counterclockwise from the positive horizontal ray to the existing ray.
Computation time

- If there are M vertices on the hull, then the computation time is MN
- First we compute N-1 angles to find the minimum, then N-2 angles, then N-3, etc.
- So, the total is (N-1)+(N-2)+…+(N-M+1)=MN-M(M-1)
- Worst case: O(N^2)
Can be extended to k dimensional space

E.g., For the 3D case it would be equivalent of sweeping a plane where the anchor is a line.
Graham’s scan method

- Basic concept: Gradually expand the “trial” hull until it reaches the whole convex hull.
- Trial Hull: all the points we have processed so far lies inside the trial hull.
- Proceed around the “trial” hull by trying to place a new point on the hull and “eliminating” previously placed points that couldn’t possibly be on the new updated “trial” hull.

- How to eliminate points?
  Use the property that on a convex hull you would never have to move clockwise from one point to another
The current “trial” hull is B M J. We try to put L into the trial hull. However, the it has to turn right (clockwise) while sweeping from ray MJ to ray JL which indicates that J can be enclosed in the updated new trial hull “BML”, so we eliminate the J from the new updated trial hull.
CCW procedure
ALGORITHM

- Start from the simplest trial hull (triangle) for the set of points given
- Add a point each time, while a right turn is made
  Eliminate the last added point in the current hull.
  Update the last added point
- Repeat the above steps until the starting node has been reached
How to decide the order of adding the point

- Sort the points first according to the key “angle”.
- Angle:
  
  Take the point with the min y always as the starting point and the N-1 points as the ending points to form N-1 rays. “Angle” is the degree swept through Counterclockwise from the positive horizontal ray to the existing ray.
The sort guarantees that each point is considered in turn as a possible hull point, because all points considered have a smaller “angle” value.
Implementation

- B M J L N P K F I E C O A H G D
- B M L N P K F I E C O A H G D
- B M L N P K F I E C O A H G D
Computation time

- Sort: $O(N \log N)$  $N$ is the number of points
- Expand the hull: Each time add a new point to the trial hull, the computation time needed to eliminate is nearly $O(1)$. And we need try to add $N-1$ points, so the computation time is $O(N) \times O(1) = O(N)$
- The dominant computation is the sort. So this algorithm takes $O(N \log N)$
INTERIOR ELIMINATION

- Idea: to eliminate most inside points
- To pick 4 points which makes a quadrilateral
  - 4 points can not be on the hull
  - Any points inside this quadrilateral can be forgotten
- Interior elimination can usually leave only $O(\sqrt{N})$ points to process
Interior elimination Using MAX X, MAX Y, MIN X and MIN Y
Interior elimination using 
max(x+y), max(x-y), max(y-x) and max(-x-y)
On the average, interior elimination method is linear

Worst case:
- When all points lie on convex hull

All points not eliminated (O($\sqrt{N}$), on average) must be processed by Package Wrapping or Graham Scan

Cost of elimination is clearly O(N)
CONCLUSION

- A convex hull is a polygon in which a line between 2 points inside the hull lies inside the polygon.
- Convex hull can be found using package wrapping, graham’s scan and interior elimination.
- Interior elimination and package wrapping can be extended to any dimensions.
- Graham’s scan has the best worst case performance-NlogN.
- Interior elimination has the best average performance.