

# Karnaugh Maps (K-Maps)

Method to simplify SOP

# Boolean Algebra

We can use the following

$$X + \bar{X} = 1$$

$$XY + XZ = X(Y + Z) \quad \text{Factoring}$$

$$X = X + X \quad \text{Duplicating}$$

## Example

$$\begin{aligned} X_1 X_0 + \bar{X}_1 X_0 + X_1 \bar{X}_0 \\ &= (X_1 + \bar{X}_1) X_0 + X_1 \bar{X}_0 \\ &= 1 \cdot X_0 + X_1 \bar{X}_0 \\ &= X_0 + X_1 \bar{X}_0 \end{aligned}$$

Galen Sasaki

EE 260 University of Hawaii

## Example

$$\begin{aligned} X_1 X_0 + \bar{X}_1 X_0 + X_1 \bar{X}_0 \\ &= X_1 X_0 + X_1 X_0 + \bar{X}_1 X_0 + X_1 \bar{X}_0 \\ &= (X_1 + \bar{X}_1) X_0 + X_1 (X_0 + \bar{X}_0) \\ &= 1 \cdot X_0 + X_1 \cdot 1 \\ &= X_0 + X_1 \end{aligned}$$

Duplicating helps

Galen Sasaki

EE 260 University of Hawaii

# Observations

Difficult to find these grouping of terms that can be factored and simplified

K-Maps help to find these groups of terms, which are called “implicants”

Let’s see if we can find implicants by just writing them down

$$\begin{array}{cccc} 00 & 01 & 10 & 11 \\ \overline{X}Y & \overline{X}Y & X\overline{Y} & XY \\ \underbrace{\hspace{1.5cm}} & \underbrace{\hspace{1.5cm}} & \underbrace{\hspace{1.5cm}} & \\ \underbrace{\hspace{3.5cm}} & & & \end{array}$$

Galen Sasaki

EE 260 University of Hawai

# Observations

$$\begin{array}{cccc} 00 & 01 & 11 & 10 \\ \overline{X}Y & \overline{X}Y & X\overline{Y} & XY \\ \underbrace{\hspace{3.5cm}} & & & \text{wrap} \\ & & & \text{around} \end{array}$$

Gray code: 00 01 11 10

Only one bit changes between neighboring numbers, including wrap around

Bits that don’t change correspond to literals that are factored out

Galen Sasaki

EE 260 University of Hawai

# Example

$$X_1X_0 + \bar{X}_1X_0 + X_1\bar{X}_0$$

$X_1X_0$

	00	01	11	10
	0	1	1	1

$X_1\bar{X}_0$

	00	01	11	10
	0	1	1	1

$X_1$	0	1
$X_0$	0	1
1	1	1