# Karnaugh Maps (K-Maps)

Method to simplify SOP



# Boolean Algebra

We can use the following

$$X + \overline{X} = 1$$

$$XY + XZ = X(Y + Z)$$ Factoring

$$X = X + X$$ Duplicating

# Example

$$X_1X_0 + \overline{X}_1X_0 + X_1\overline{X}_0$$

$$= \left(X_1 + \overline{X}_1\right)X_0 + X_1\overline{X}_0$$

$$= 1 \cdot X_0 + X_1\overline{X}_0$$

$$= X_0 + X_1\overline{X}_0$$



# Example

$$X_1X_0 + \overline{X}_1X_0 + X_1\overline{X}_0$$

$$= X_1X_0 + X_1X_0 + \overline{X}_1X_0 + X_1\overline{X}_0$$

$$= \left(X_1 + \overline{X}_1\right)X_0 + X_1(X_0 + \overline{X}_0)$$

$$= 1 \cdot X_0 + X_1 \cdot 1$$

$$= X_0 + X_1$$

Duplicating helps

# Observations

Difficult to find these grouping of terms that can be factored and simplified

K-Maps help to find these groups of terms, which are called “implicants”

Let’s see if we can find implicants by just writing them down

| 00 | 01 | 10 | 11 |
| --- | --- | --- | --- |
| $\overline{X}\,\overline{Y}$ | $\overline{X}Y$ | $X\overline{Y}$ | $XY$ |



# Observations

| 00 | 01 | 11 | 10 |
| --- | --- | --- | --- |
| $\overline{X}\,\overline{Y}$ | $\overline{X}Y$ | $XY$ | $X\overline{Y}$ |

wrap around

Gray code: 00 01 11 10

Only one bit changes between neighboring numbers, including wrap around

Bits that don’t change correspond to literals that are factored out

# Example

$$X_1X_0 + \overline{X}_1X_0 + X_1\overline{X}_0$$