Propagation of Signals in Optical Fiber

Outline
- Geometric approach
- Wave theory approach
- Loss and Bandwidth

Geometric Approach
Ray theory or geometrical optics:
- Light behaves like rays
- Index of refraction: speed of light
- Snell's Law: how light bends
- Modal dispersion: effect on bit-rate x distance
- Fiber types
  - Step index
  - Graded index
  - Single mode

Refraction
speed of light = c/n

Snell's Law: \( n_1 \times \sin \theta_1 = n_2 \times \sin \theta_2 \)
Critical angle = \( \sin^{-1} \frac{n_2}{n_1} \) then \( \theta_2 = \pi/2 \)
total internal reflection

How do we use this?
- Contain the light down the fiber
- Make sure light pulses keep their shape
**Definitions**

- **Numerical Aperature (NA)**: \( n_0 \sin \theta_0^{\text{max}} \)
  - A measure of light gathering
  - \( \Delta = \frac{n_1 - n_2}{n_1} \) (recall \( n_1 = \text{core}, n_2 = \text{cladding} \))
  - \( \Delta \) is typically small, e.g., 0.01
  - For small \( \Delta \), \( NA = n_1 \sqrt{2\Delta} \)

Example: \( n_1 = 1.5 \) for silica and \( \Delta = 0.01 \)
NA is about 0.212. Thus \( \theta_0^{\text{max}} = 12 \) degrees

**Modal Dispersion**

- Center: \( T_0 = \frac{L}{n_1/c} \)
- Critical angle: \( T_s = \frac{L}{n_1^2/cn_2} \)

- Bit-rate distance product \( BL \) (Mb/s)-km,
  - \( B \text{ Mbps over } L \text{ km} \). (Loss is the main limitation)
  - Product is usually constrained by a constant
  - Modal dispersion constraint on the product
    \[ BL < \frac{1}{2} \frac{n_1}{n_2^2} \frac{c}{\Delta} \]

Rearranging the bit rate inequality:

\[ \frac{1}{2} \frac{n_1}{n_2^2} \frac{c}{\Delta} < BL \]

Step index fiber
Graded-Index Fiber

- Index changes more smoothly from core to cladding in a quadratic way
- Different $\delta T$

\[ \Delta = 0.01, \ n_1 = 1.5 \]

Graded index

\[ BL < \frac{4c}{n_1 \Delta} = 8 \text{ (Gb/s)-km} \]

Step index fiber

\[ BL < \frac{1}{2} \frac{n_2}{n_1^2} \Delta = 10 \text{ (Mb/s)-km} \]

Geometric Optics Approach

- Okay for multi-mode because core radius $a$ is 250-100 $\mu$m, larger than wavelengths
- Single-mode fiber has smaller core
  - Diffraction effect
  - Refocus spreading light beam
  - Explained by wave theory

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Wave Theory

- Review of electromagnetic basics
- Electric and magnetic fields
- Five characteristics of the medium
- Wave equations
- Fiber modes
- Polarization
- Light propagation in wave guides

Review of EM

**Electric Field**

\[ qE = F \]

\[ F = \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_2}{r^2} \]

\[ E = \frac{F}{q_0} = \frac{1}{4\pi \varepsilon_0} \frac{q}{r^2} \]

**Magnetic Field**

\[ F = qv \times B \]

\[ F = qvB \sin \theta \]
Wave Theory

- \( E(r,t) \): electric field
- \( H(r,t) \): magnetic field

Fourier Transforms:

\[
\tilde{E}(r,\omega) = \int_{-\infty}^{\infty} E(r,t) \exp(i\omega t) dt
\]

Wave Theory

- \( P \): Induced electric polarization (or polarization)
- \( M \): Magnetic polarization

\[
D = \varepsilon_0 E + P
\]
\[
B = \mu_0 (H + M)
\]

\( \varepsilon_0 \): permittivity of vacuum
\( \mu_0 \): permeability of vacuum

Relationship between P and E

- Relationship between \( P \) and \( E \) affects dispersion and nonlinearities
- Five affects and their affect on \( P \) and \( E \)
  - Locality of response
  - Isotropy
  - Linearity
  - Homogeneity
  - Losslessness

Locality of Response

For any \( r_1 \), \( P(r) \) at \( r_1 \) is only dependent on \( E(r_1) \)
Approximately true for silica 500-2000 nm

Example of nonlocal response:
\[
P(r_1) \text{ is dependent on } \int_{V(r_1)} E(x,y,z) dx dy dz
\]

Isotropy

- Refractive index is the same in all directions
- This means \( E \) and \( P \) have the same orientation
- Silica is isotropic
- Perfectly cylindrical fiber is isotropic
- Imperfect fiber is not isotropic, i.e., birefringent

Linearity

\[
P(r,t) = \varepsilon_0 \int \chi(r,t-u) E(r,u) du
\]
\[
\tilde{P}(r,\omega) = \varepsilon_0 \tilde{\chi}(r,\omega) \tilde{E}(r,\omega)
\]

Linear susceptibility

- \( P \) may have a delayed response to \( E \)
- Chromatic dispersion

Linearity is true for silica at moderate powers and bit rates
Homogeneity

\[ \chi(t, r) = \chi(t), \]

i.e., electromagnetic properties are independent of position.

Silica is homogeneous, but fiber is not.

\[ n^2(\omega) = 1 + \chi(\omega) \]

Fiber has core with different index of refr. than cladding.

Losslessness

Silica has zero loss, kind of.

For the time being, we'll assume all five properties.

Wave Equations

From Maxwell's Equations

\[ \nabla \times \vec{E} + \frac{\omega^2 n^2(\omega)}{c^2} \vec{E} = 0 \]

\[ \nabla \times \vec{H} + \frac{\omega^2 n^2(\omega)}{c^2} \vec{H} = 0 \]

\[ \nabla \times \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \vec{E} = \nabla \times \vec{H} = 0 \]

This dictates the EM waves that go through the medium.

What do the Wave Equations do for us?

• The wave equations tell us how signals travel down a medium, like a fiber.

Fiber mode: solutions for

• wave equations for the core
• wave equations for the cladding
• boundary conditions between cladding and core

Fiber Modes

• Suppose
  – \( z \) = direction of electromagnetic wave
  – fiber properties are independent of \( z \)

\[ \text{step index fiber} \quad \text{z} \]

• Then electric and magnetic fields of a fiber mode are dependent on \( z \) as \( \exp(i\beta z) \), a sinusoid propagation constant

Propagation Constant \( \beta \)

• Propagation constant \( \beta \) is \( \omega n / c = 2\pi n/\lambda \).

• Propagation constant is \( kn \), where \( k = 2\pi / \lambda \), the wave number

• \( kn_2 < \beta < kn_1 \)

\[ \text{cladding} \quad \text{core} \]

neff = \( \beta/k \)

In general \( \beta \) is a function of \( \omega \).
Normalized Propagation Constant

\[ b = \frac{\beta^2 - k^2 n_2^2}{k^2 n_1^2 - k^2 n_2^2} = \frac{n_{\text{eff}}^2 - n_2^2}{n_1^2 - n_2^2} \]

\[ b(V) \approx (1.1428 - 0.9960/V)^2 \]

Single Mode

Conditions for single mode:

\[ V := \frac{2\pi}{k} a \sqrt{n_1^2 - n_2^2} < 2.405 \]

Typical values:

\[ a = 4000\,\text{nm}, \quad \Delta = (n_1-n_2)/n_1 = 0.003, \quad V = 2 \, @ 1550\,\text{nm} \]

Multimode

For large V (multimode),

# modes is approximately \( V^2/2 \)

Multimode \( \rightarrow \) several hundred modes

Polarization

Let’s take a look at the signal on a single mode fiber

\[ \nabla^2 \tilde{E} + \frac{\omega^2 n_1^2 (\omega)}{c^2} \tilde{E} = 0 \]

2 linearly indep solutions

\[ \tilde{E}(r,t) = \tilde{E}_1 \hat{e}_x + \tilde{E}_2 \hat{e}_y \]

Polarization

Properties of the electric field on a single mode fiber

- Linearly polarized: its direction is constant with time
- Solution = \( \alpha_1 \) Solution1 + \( \alpha_2 \) Solution2
- It is transverse since it has “no” component along the direction of propagation (actually very small)
- Polarization Mode Dispersion (PMD)
  If the fiber is not perfectly circular, the \( x \) and \( y \) components have different speeds

\[ E_z = 2\pi J_z(x,y) \exp(i\beta z) \]

\[ E_x (E_y) = 2\pi J_x (x,y) \exp(i\beta z) \]

\[ J_x (J_y) \text{ are dependent on } (x,y) \text{ only through} \]

\[ \rho = \sqrt{x^2 + y^2} \]

\[ \tilde{E}(r,\omega) = 2\pi J(x,y) \exp(i\beta(\omega)z)\hat{e}(x,y) \]

\[ J(x,y) = \sqrt{J_x (x,y)^2 + J_y (x,y)^2} \]
Light Propagation in Dielectric Waveguides

Slab rather than a cylinder

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Loss and Bandwidth

• Attenuation formula
• Loss mechanisms
• Bandwidth

Loss Formula

\[ P_{\text{out}} = P_{\text{in}} \exp(-\alpha L) \]

Length of medium

Customary to express the loss in dB/km

\[ \text{\text{-\ensuremath{\frac{\text{dB}}{\text{km}}}}} = 10 \log_{10}(\frac{P_{\text{out}}}{P_{\text{in}}}) \] dB/km

Different from the \( \alpha \) above

\[ \alpha_{\text{dB}} = (10 \log 10 \phi) \alpha \approx 4.343 \alpha \]

Example

fiber

0.25 dB/km

How far can a signal go?

Typically, a signal can suffer 20-30 dB loss before being amplified or attenuated

The signal can go 20/0.25 to 30/0.25 km

Loss Mechanisms

• Material Absorption
  – due to impurities in silica
  – negligible for 800-1600 nm
• Rayleigh Scattering
  – dominant loss mechanism
  – due to fluctuations in the density of the medium at the microscopic level
  – loss coefficient \( \alpha_{\text{R}} = \frac{A}{\lambda^4} \)
Fiber Attenuation

Wavelength $\lambda$ (µm)

- 800nm
- 1310nm
- 1550nm

Loss (dB/km)

- 2.5 dB/km
- 0.4 dB/km
- 0.25 dB/km

Bandwidth

$$f = \frac{c}{\lambda}$$

$$\Delta f \approx \frac{(c \Delta \lambda)}{\lambda^2}$$

35 THz of bandwidth in single mode fiber